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Multi parametric analysis of state feedback control of permanent magnet synchronous machine

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Abstract

This paper present a method for controlling the stability and bifurcation of the permanent magnet synchronous machine. This approach employs the idea used in computing the transition variety sets of constrained bifurcations to find the stability boundaries of an equilibrium points in parameter space. Then, a feedback control method is used to control the machine and to detect a different bifurcation points. A theory analysis is used to explain the evolution of the critical points in the parameter space. Thus, the paper deals with the analytical representation of bifurcation of the machine.

Key words: Bifurcation surfaces, boundary stability feedback control, pole placement, PMSM.

I-Introduction

The investigation of bifurcations in electric motors is a field of active research due to its direct applications in many areas, such as, industrial machinery, electrical locomotives and electrical submersibles thruster drives,[7], [13], [14]. The appearing of bifurcations phenomena in machine drives allows us to study the real stability of system. As parameters change, the equilibrium point can lose its stability in such a way that a pair of complex conjugate eigenvalues of the Jacobien matrix of the system crosses the imaginary axis of the complex plane, from the left-half plane to the right- half plane, such that the machine may start oscillating with small amplitude. This phenomena of loss of stability is associated with a hopf bifurcation where a periodic oscillation emerges from a stable equilibrium point EP and another small perturbation on the machine parameters provokes the onset of growing oscillation. Then, the dynamics behavior of permanent magnet synchronous motor is studied by means of modern nonlinear theories such as bifurcation and chaos, [3], [5], [7]. In reality, the dynamic evolution of natural systems displays a large variety of qualitatively different long-trem behaviors. The dynamic of machine can be stationary, periodic, quasi periodic and chaotic [2]. Many works is f ocused on analysis of dynamic properties of controlled drive of permanent magnet synchronous machine and also on bifurcation of steady states and possible occurrence of chaotic behavior, [13], [14]. Controlling such instabilities is the mean concern of many researchs, [6], [8], [11]. These study show how to stabilize the system, while having a satisfactory performance, even in the case when some of the motor parameters were uncertain, [5], [2]. The critical phenomena are defined by the position of the boundaries of attraction of the equilibrium point. The dynamical surface evolution of the critical point is corresponding to the case in which one of the eigenvalues of the jacobien matrix of the linear approximation of the nonlinear system cross the imaginay axis of the complex plan, [1], [8], [9], [10], [11].

The paper is organized as follows: the first section concern to explain the mathematical model of the machine and a feedback control of the system is studied in order to determine the global model of the system. In section2, a Pole placement method will be detailed where a necessary condition is analyzed for the appearing of the stability in first step and a different bifurcation points in the second step. Another section concern to a numerical analysis of the existence of critical point.

II-Mathematical model of PMSM drive system and preliminaries

1- Mathematical preliminaries

The dynamic of permanent magnet synchronous machine can be modeled by parameter dependent differentials equations of the form.

$\sum: \dot{x} = f(x,\mu) \quad f: \mathbb{R}^8 \to \mathbb{R}^3$

$x \in X \subset \mathbb{R}^3,\ \mu \in P \subset \mathbb{R}^5$

For the constrained system $\sum$, define the set of all equilibrium points to be E_μ and let $E_\mu{}^s$ denote the set of all stable equilibria defined as

$E_\mu = \{(x^e,\mu) \in X \times P: F(x^e) = 0\}$

$E_\mu{}^s = \{(x^e,\mu) \in E_\mu: A(x^e) = \frac{\partial f_i}{\partial x_i}, i = 1 \dots 3,$ in nonsingular and all eigenvalues of $A(x^e)$ have negative real parts}. Note that the jacobien$A(x^e) = \frac{\partial f_i}{\partial x_i}$ of the fonctuon f in the coordinates x in nonsingular for all $(x^e, \mu) \in E_\mu{}^s$ and therefore, by the implicit function theorem, the equations $f(x^e, \mu) = 0$ can theoretically be solved uniquely for x^e as functions of parameter μ, locally near any equilibrium point in $E_\mu{}^s$. Hence $E_\mu{}^s$ is a $\mu - dimensional$ submanifold embedded in $E_\mu \complement X \times P$.

Now, studding of the dynamical behavior of machine need to define a three sets which constitute the critical surface boundary (bifurcation) correspond to specific conditions on the eigenvalues of the system. As a point on E_μ approaches S_{SN} some eigenvalues diverge to infinity. The equilibrium has zero eigenvalues in the set S_{SN} and has a conjugate purely imaginary eigenvalues for points in the set S_H. Moreover, generically these bifurcations constitute a dense subset of the critical surface boundary for the system and its defined by:

$$S_{SN} = \{(x^e, \mu) \in E_\mu : \det(A(x^e)) = 0\}$$

$$S_H = \{(x^e, \mu) \in E_\mu : \det(A(x^e)) \neq 0, \det(R_{n-1}(A(x^e)) = 0\}$$

$$S_{SNH} = \{(x^e, \mu) \in E_\mu : \det(A(x^e)) = \det(R_{n-1}(A(x^e)) = 0,\}$$

Where

$$R_{n-1} = \begin{pmatrix} a_1 & a_3 & a_5 & \cdots & a_{2n-3} \\ 0 & a_2 & a_4 & \dots & a_{2n-4} \\ 0 & a_1 & a_3 & \dots & a_{2n-5} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{n-1} \end{pmatrix}$$

2-Model description of the machine

The mathematical model of PMSM with the smooth air gap can be described as follows:

$$\Sigma: \begin{cases} \dot{x}_1 = f_1(x,\mu) = \frac{-R_s}{L} x_1 + n_p x_2 x_3 + \frac{1}{L} v_d \\ \dot{x}_2 = f_2(x,\mu) = \frac{-R_s}{L} x_2 - n_p x_1 x_3 - \frac{n_p \phi_f}{L} x_3 + \frac{1}{L} v_q \\ \dot{x}_3 = f_3(x,\mu) = \frac{n_p m \phi_f}{2J} x_2 - \frac{f}{J} x_3 - \frac{1}{J} T_L \end{cases} \quad (1)$$

3-State feedback controller of PMSM

The linearized model of system around an equilibrium point x^e of our machine is described by[4]

$$\dot{x} = F(x,u) = Ax + Bu + ET_L$$

With: $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} i_d \\ i_q \\ \Omega \end{pmatrix}$ is the states variables of the system. $u = \begin{pmatrix} v_d \\ v_q \\ 0 \end{pmatrix}$ is the vector of input and $E = \begin{pmatrix} 0 \\ 0 \\ \frac{-1}{J} \end{pmatrix}$ is a constant vector of the load input

$A(x^e) = \begin{bmatrix} \frac{-R_s}{L} & n_p x^e{}_3 & n_p x^e{}_2 \\ -n_p x^e{}_3 & \frac{-R_s}{L} & -n_p(x^e{}_1 + \frac{\phi_f}{L}) \\ 0 & \frac{n_p m \phi_f}{2J} & \frac{-f}{J} \end{bmatrix}$ is the jacobien matrix of mathematical model of machine.

$B = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix}$ is the constant matrix of the machine input. Now, We wish to stabilize this system about some controlled equilibrium state x^e. Let u^e be an input that achives the desired controlled equilibrium state x^e, that is, $F(x^e, u^e) = 0$. The linear static state feedback controllers, that is, at each instant t of time the current control input u(t) depends linearly on the current state x(t).

The control input can be described by: $u = Kx + r$

$$u = Kx + r = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ 0 \end{bmatrix}$$

Where K is a constant matrix mxn, sometimes called a state feedback gain matrix,[4]. When our system is subject to such a controller, its behavior is governed by

$$\dot{x} = (A + BK)x + Br + ET_L$$

Then, The model of system will be described by the following differentials equations:

$$\begin{cases} \dot{x}_1 = \mu_1 x_1 + n_p x_2 x_3 + \mu_2 x_2 + \mu_3 x_3 + \frac{v_{dref}}{L} \\ \dot{x}_2 = \mu_1 x_2 - n_p x_1 x_3 + \mu_4 x_1 + \mu_5 x_3 + \frac{v_{qref}}{L} \\ \dot{x}_3 = c_1 x_2 + c_2 x_3 - \frac{1}{J} T_L \end{cases} \quad (2)$$

with

$\mu_1 = -\frac{R_s + k_{11}}{L}$, $\mu_2 = \frac{k_{12}}{L}$, $\mu_3 = \frac{k_{13}}{L}$, $\mu_4 = \frac{k_{21}}{L}$, $\mu_5 = -\frac{n_p \phi_f + k_{23}}{L}$ $c_1 = \frac{n_p m \phi_f}{2J}$, $c_2 = -\frac{f}{J}$

The machine operate in permanent regime when,[4]:

$$x_2{}^e = \frac{2f}{pm\,\phi_f} x_3{}^e + \frac{2}{pm\,\phi_f} T_L$$

$$v_d{}^e = R_s x_1{}^e - pL_q x_2{}^e x_3{}^e$$

$$v_q{}^e = R_s x_2{}^e + pL_d x_1{}^e x_3{}^e + p\phi_f x_3{}^e$$

Now choosing the operation point of machine $x^e = (x_1{}^e, x_2{}^e, x_3{}^e)$ we can compute the input voltage $u^e = (v_d{}^e,\ v_q{}^e)$ and consequently the reference input $r = (v_{dref}, v_{qref})$ will be expressed by:

$$r = u^e - K\,x^e$$

Now, the objective is to find the constant of matrix K such that the linearized model around the operating point x^e has the following desired eignevalues,[12]:

$$P = [-10, -5 + 80i, -5 - 80i]$$

Using the matlab function $K = place(A, B, P)$, we can determine the value of the matrix K at the equilibrium point EP: $x^e = (11.54, 6.17, 43.38)$ in which the load torque$T_L = 5N.m$, and the inputs voltage on machine $(v_d{}^e,\ v_q{}^e) = (5.0154, 47.349)$.

Then, the state feedback matrix will be defined by:

$$K = \begin{bmatrix} -1.090 & 1.431 & 0.203 \\ -1.431 & -1.090 & -0.016 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, using the elements of the state feedback matrix K, we may compute the references inputs voltages that it's fixed at $(v_{dref}, v_{qref}) = (-0.0414, 71.2821)$.

And then the parameters of the machine will have the follows values;

$\mu_1 = -208.1818$, $\mu_2 = 130.0909$, $\mu_3 = 18.4545$, $\mu_4 = -130.0909$, $\mu_5 = -50.5455$

III-Bifurcation surfaces expressions

The stability of the dynamical machine around an equilibrium point will be controlled by the eigenvalue of the follows matrix of the machine model describing by equations (2): $A + BK =$

$$\begin{bmatrix} \mu_1 & n_p x^e{}_3 + \mu_2 & n_p x^e{}_2 + \mu_3 \\ -n_p x^e{}_3 + \mu_4 & \mu_1 & -n_p x^e{}_1 + \mu_5 \\ 0 & c_1 & c_2 \end{bmatrix}$$

Taking the equilibrium point at the origin of machine $x^e = (0,0,0)$, the jacobien matrice will be expressed by

$$A + BK = \begin{bmatrix} \mu_1 & \mu_2 & \mu_3 \\ \mu_4 & \mu_1 & \mu_5 \\ 0 & c_1 & c_2 \end{bmatrix}$$

So, when the eigenvalues of $A_{cl} = (A + BK)$ have a negative real parts, the closed loop of nonlinear system is asymptotically stable about x^e.

The characteristic polynome of the system is defined by

$$P(\lambda) = det(\lambda I - A - BK)$$

The characteristic polynome of the jacobien matrix is defined by:

$$P(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$$

Which:

$$a_1 = -tr(A_{cl}) = -\left(\frac{\partial f_1}{\partial x_1} + \frac{k_{11}}{L}\right) - \left(\frac{\partial f_2}{\partial x_2} + \frac{k_{22}}{L}\right) - \frac{\partial f_3}{\partial x_3} = -2\mu_1 - c_2$$

$$a_2 = \begin{vmatrix} \mu_1 & \mu_2 \\ \mu_4 & \mu_1 \end{vmatrix} + \begin{vmatrix} \mu_1 & \mu_3 \\ 0 & c_2 \end{vmatrix} + \begin{vmatrix} \mu_1 & \mu_5 \\ c_1 & c_2 \end{vmatrix} = \mu_1{}^2 - \mu_2\mu_4 + 2c_2\mu_1 - c_1\mu_5$$

$$a_3 = \det(A_{cl}) = -\begin{vmatrix} \mu_1 & \mu_2 & \mu_3 \\ \mu_4 & \mu_1 & \mu_5 \\ 0 & c_1 & c_2 \end{vmatrix} = \mu_4(c_2\mu_2 - c_1\mu_3) - \mu_1(c_2\mu_1 - c_1\mu_5)$$

1-Boundary stability of equilibrium point

Using the Routh-Hurwitz matrix R_{n-1}, we can explain the necessary and the sufficient conditions of stability and then all the roots of the polynomial have negative real parts, that given by:

$$\Delta_i(\mu) > 0, i = 1 \dots 3$$

Where $\Delta_i(\mu)$ are called principal minors of the Hurwitz arrangement of order n

The first condition of stability is that all the coefficients of the characteristic polynomial are positive $a_i > 0$.

$$a_1 = -(2\mu_1 + c_2) > 0$$
$$a_2 = \mu_1{}^2 - \mu_2\mu_4 + 2c_2\mu_1 - c_1\mu_5 > 0$$
$$a_3 = \mu_4(c_2\mu_2 - c_1\mu_3) - \mu_1(c_2\mu_1 - c_1\mu_5) > 0$$

The second condition of stability need to calculate the minors of the Hurwitz arrangement.

$$\Delta_1(\mu) = |a_1| = -(2\mu_1 + c_2) > 0$$
$$\Delta_2(\mu) = \begin{vmatrix} a_1 & 1 \\ a_3 & a_2 \end{vmatrix} = a_1a_2 - a_3 > 0$$
$$\Delta_3(\mu) = \begin{vmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ 0 & 0 & a_3 \end{vmatrix} = (a_1a_2 - a_3)a_3 = a_3\Delta_2(\mu) > 0$$

In order to study the dynamical behavior of system, we need to explain the boundaries of the domain of attraction of the stability of system. Then, a critical bifurcation surface expression will be detailed, [10]. Thus the domain of attraction of the equilibrium point $x^e = (x_1{}^e, x_2{}^e, x_3{}^e)$ is defined by the three inequalities

$$\Delta_1(\mu) = -(2\mu_1 + c_2) > 0$$
$$\Delta_2(\mu) = a_1a_2 - a_3 > 0$$
$$\Delta_3(\mu) = (a_1a_2 - a_3)a_3 = a_3\Delta_2(\mu) > 0$$

Thus a necessary condition of the parameter μ will be defined by

$$\mu_1 < \frac{-c_2}{2} = 0.0083$$
$$\mu_4 > \frac{2\mu_1{}^3 + 4c_2\mu_1{}^2 - (c_1\mu_5 - 2c_2{}^2)\mu_1 - c_1c_2\mu_5}{c_1\mu_3 + 2\mu_2\mu_1} = l_H(\mu)$$
$$\mu_4 > \frac{\mu_1(c_2\mu_1 - c_1\mu_5)}{c_2\mu_2 - c_1\mu_3} = l_{SN}(\mu)$$

Note that $l_H(\mu)$ is defined for $2\mu_1\mu_2 + c_1\mu_3 \neq 0$. Then the critical values of parameter μ_2 are defined by $\mu_{2c} = \frac{-c_1\mu_3}{2\mu_1}$.

Thus, the parametric function $l_{SN}(\mu)$ has a critical point at $\mu_{2c} = \frac{c_1\mu_3}{c_2}$, and the boundaries of the domain of attraction are defined by the two planes

$$\wp_1 \equiv \left\{\mu \in \mathcal{R}^5 \middle| \mu_1 = \mu_{1c} = \frac{-c_2}{2}\right\}$$
$$\wp_2 \equiv \{\mu \in \mathcal{R}^5 | \Delta_2(\mu) = a_1a_2 - a_3 = 0\}$$

And the saddle plan which expressed by the follow equation

$$\wp_3 \equiv \{\mu \in \mathcal{R}^5 | \Delta_3(\mu) = (a_1a_2 - a_3)a_3 = a_3\Delta_2(\mu) = 0\}$$

Then, the three planes $\wp_1, \wp_2, \wp_3$ define the boundaries of the domain of attraction and they are called the critical bifurcation surfaces which we will detailed in the next section

2- Hopf bifurcation surface:

Taking the follow case, in which a Hopf bifurcation point is appeared when the eigenvalues take the follow form,[24]:

$$\lambda_{1,2} = \pm j\sigma_0$$

$$\lambda_3 = \beta_1$$

Thus, the desired characterisitc equation is defined by

$$p_h(\lambda) = \lambda^3 - \beta_1\lambda^2 + {\sigma_0}^2\lambda - \beta_1{\sigma_0}^2$$

Then, $p(\lambda) = p_h(\lambda)$ when the coeficients of the two characteristic polynomials are equal and then the real and imaginary parts of eigenvalues will be expressed by.

$$\beta_1 = -a_1 = 2\mu_1 + c_2$$

$${\sigma_0}^2 = a_2 = {\mu_1}^2 - \mu_2\mu_4 + 2c_2\mu_1 - c_1\mu_5$$

$$a_3 = -\beta_1{\sigma_0}^2 \qquad (3)$$

Now, replacing respectively β_1 and ${\sigma_0}^2$in the expression (3) by $-a_1$ and a_2 , The conditions for the appearing of the hopf bifurcation are defined by $\Delta_2(\mu) = a_1a_2 - a_3 = 0$, in which we may determine $\mu_4 = l_H(\mu_1, \mu_5)$ And then the expression of the imaginary parts of the eigenvalues λ_1 and λ_2 will be expressed by

$$\sigma_0(\mu)^2 = a_2 = {\mu_1}^2 - l_H(\mu) + 2c_2\mu_1 - c_1\mu_5$$

thus

$$\sigma_0(\mu) = \sqrt{\frac{c_1\mu_3{\mu_1}^2 - (\gamma_1 + c_1\mu_5\mu_2)\mu_1 + \gamma_2\mu_5}{c_1\mu_3 + 2\mu_2\mu_1}} = \sqrt{\frac{p(\mu)}{q(\mu)}}$$

with

$$\gamma_1 = 2{c_2}^2\mu_2 - 2c_2c_1\mu_3$$

$$\gamma_2 = c_1c_2\mu_2 - {c_1}^2\mu_3$$

$\lambda_{1,2}$ are purely imaginaire if the follows conditions are satisfied.

$q(\mu) > 0$ when $\mu_1 > \frac{-c_1\mu_3}{2\mu_2}$ with $\mu_2 \neq 0$

And $p(\mu) > 0$. Now, equating $p(\mu)$, then it's discriment is $\Delta = {\mu_5}^2 + \alpha_1\mu_5 + \alpha_2$. With $\alpha_1 = \frac{2\gamma_1\mu_2 - 4\gamma_2\mu_3}{{\mu_2}^2}$ and $\alpha_2 = \frac{{\gamma_1}^2}{(c_1\mu_2)^2}$

Now, look that Δ as a function depend of μ_5, then we will search the roots of Δ;

$\Delta' = {\alpha_1}^2 - 4\alpha_2 = 0$ if ${\alpha_1}^2 = 4\alpha_2$

And then

$\mu_{5c}(\mu_2, \mu_3) = \frac{-\alpha_1}{2} = \frac{2\gamma_2\mu_3 - \gamma_1\mu_2}{{\mu_2}^2}$ .Thus, taking $\Delta = 0$, $\mu_{1c}(\mu_2, \mu_3) = \frac{\gamma_1 + c_1\mu_{5c}\mu_2}{2}$ and for $\mu_1 < \mu_{1c}$

$\sigma_0(\mu) \neq 0$

Then, the eigenvalues $\lambda_{1,2}$ is purely imaginary and consequently a hopf bifurcation point is occuring.

3-Bagdanov-Takens bifrucation

Taking the case in which $\sigma_0(\mu) = 0$, the two eigenvalues $\lambda_{1,2}$ are equal to zero, then this is a condition for the appearing of the bifurcation of codimension 2 that it's called a Bagdanov-Takens bifurcation BTand it will be designed by

$$S_{BT1} = \{(\mu, x) \to \sigma_0(\mu) = 0 | \mu_1 = \frac{\gamma_1 + c_1\mu_{5c}\mu_2}{2}\}$$

Another case in which a Bagdanov-Takens bifurcation is detected when

$$S_{BT2} = \{(\mu, x) \to \sigma_0(\mu) = 0 | \mu_1 = \frac{\gamma_1 + c_1\mu_2\mu_{5c} \pm \sqrt{\Delta}}{2} : \Delta > 0\}$$

4-Neutrale saddle bifurcation

This type of bifurcation occur the two eigenvalues $\lambda_{1,2}$ are real and satisfied the follow condition

$$\lambda_1 + \lambda_2 = 0$$

The first condion in which $\lambda_{1,2}$ are real is verified if

$\frac{p(\mu)}{q(\mu)} < 0$ when $p(\mu) < 0$

Look that to satisfy this conditon we need to vary one of the fourth parameter (μ_2, μ_3) in which the roots of $p(\mu)$ and Δ as a function of μ_2, μ_3.

$$\mu_{5c}(\mu_2, \mu_3) = \frac{2\gamma_2\mu_3 - \gamma_1\mu_2}{{\mu_2}^2}$$

$$\mu_{1c}(\mu_2, \mu_3) = \frac{\gamma_1 + c_1\mu_{5c}\mu_2}{2}$$

The surface S_H play an important role in the formation of the chaotic region. The chaotic attractor is formed from a pair of limit cycles which emerge from the nontrivial steady state in the hopf bifurcation and undergo a homoclinic bifurcation with the trivial steady state.

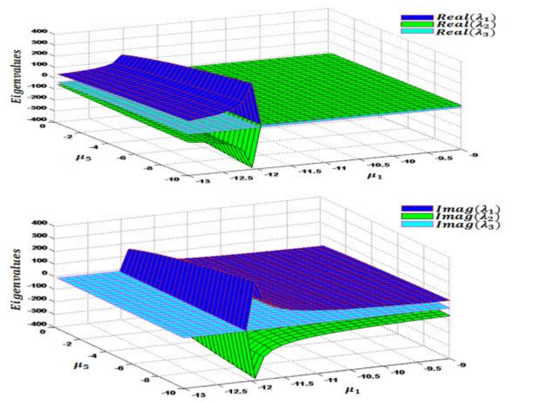
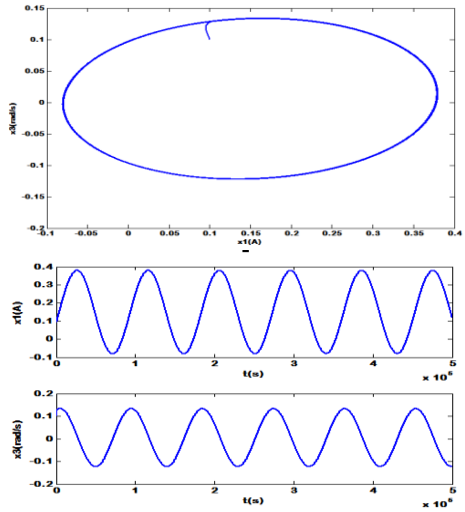
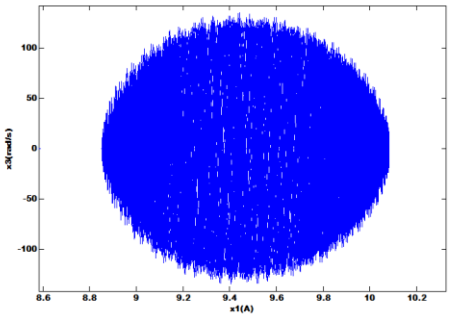


Figure1: Dynamics of eigenvalues in the parameter plane $\mu_1 - \mu_5$

Figure 1 explain the oscillatory dynamic of the PMSM drive near a hopf bifurcation surface in which the eigenvalues λ_1 and λ_2 are purely imaginary. Using the figure 2, one may verify the oscillatory dynamic of machine in phase plane and for $\mu_3 = 0.024888888$. Decreasing μ_3 to 0.0243, dynamic behavior of PMSM will be chaotic



$\mu_3 = 0.024888888$



$\mu_3 = 0.0243$

Figure 2: dynamic behavior of PMSM in phase plan

5-Saddle-node surface:

Now, for the saddle node bifurcation, taking the follow situation of the eigenvalues

$$\lambda_{1,2} = \beta_2 \pm j\sigma_0$$

$$\lambda_3 = 0$$

And using the conditions for the appearing of this critical point, then

$$a_3 = \mu_4(c_2\mu_2 - c_1\mu_3) - \mu_1(c_2\mu_1 - c_1\mu_5) = 0$$

thus

$$\mu_4 = l_{SN}(\mu_1,\mu_5) = \frac{\mu_1(c_2\mu_1 - c_1\mu_5)}{(c_2\mu_2 - c_1\mu_3)}$$

And Then, the real and the imaginary parts of the eigenvalue will be expressed by

$$\beta_2 = \frac{-a_1}{2},\ \sigma_0 = \sqrt{\frac{4a_2 - a_1{}^2}{4}}$$

Now,look that $\beta_2 < 0$, when $\mu_1 < \frac{-c_2}{2} = 0.0083$.

For another hand, the eigenvalues $\lambda_{1,2}$ are real if $4a_2 - a_1{}^2 < 0$. in which

$$\mu_4 > l_3(\mu_1,\mu_5) = \frac{4(c_2\mu_1 - c_1\mu_5) - c_2{}^2}{4\mu_2}$$

So the evolution of the eigenvalues are described by the surfaces

$$S_{\lambda_{1,2}}{}^{(1)} = \{\mu = (\mu_1,\mu_3) |\ \lambda_{1,2} = \frac{2\mu_1 + c_2 \pm \sqrt{4a_2 - a_1{}^2}}{2} : 4a_2 - a_1{}^2 < 0\}$$

$$S_{\lambda_{1,2}}{}^{(2)} = \{\mu = (\mu_1,\mu_3) |\ \lambda_{1,2} = \frac{2\mu_1 + c_2 \pm j\sqrt{4a_2 - a_1{}^2}}{2} : 4a_2 - a_1{}^2 > 0\}$$

Then, the saddle node bifurcation are caracterized by the follow surface

$$S_{SN} \equiv \left\{\mu = (\mu_1,\mu_3) \middle| \Delta_3(\mu) = (a_1a_2 - a_3)a_3 = a_3\Delta_2(\mu) = 0 : \frac{-a_1}{2} \neq 0\right\}$$

This equality defines the structure of bifurcations on the saddle in the following way. On the saddle all three of each eigenvalue are not more than zero, since a saddle is the boundary of the domain of asymptotic stability and, moreover, on the saddle we have one real eigenvalue and two complex conjugate eigenvalues.

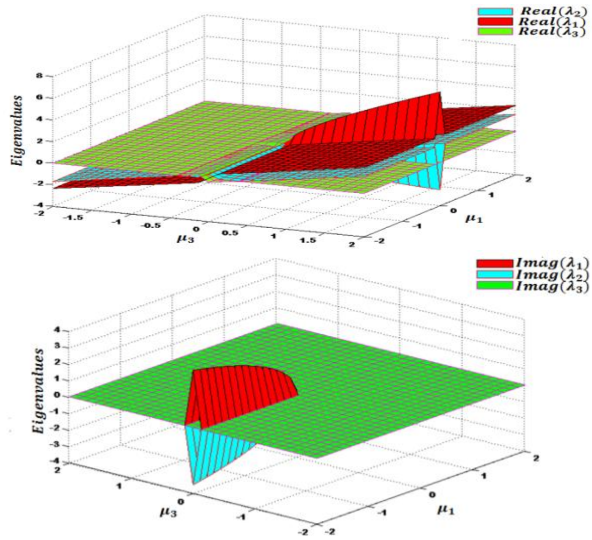
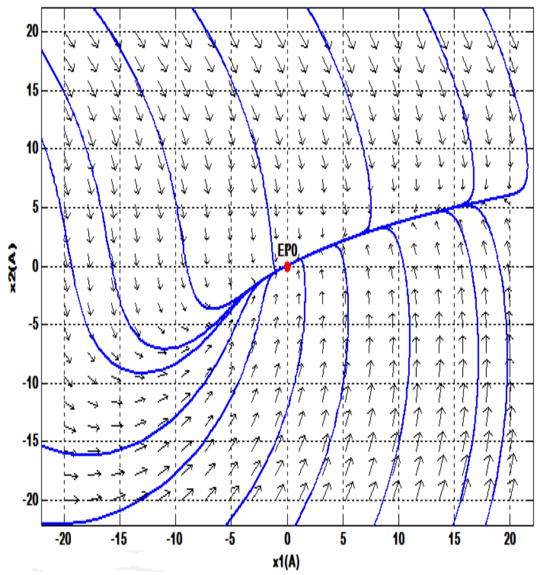
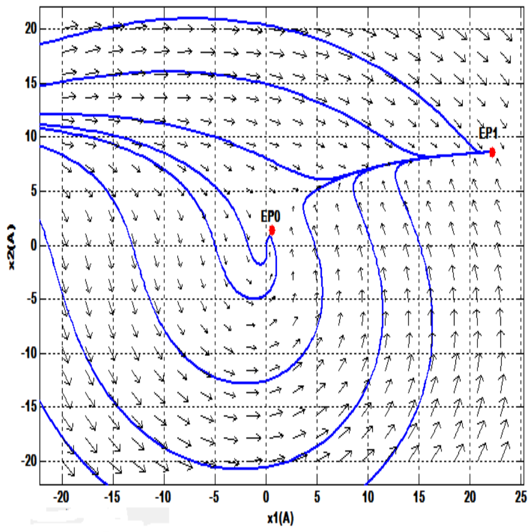


Figure3: Evolution of eigenvalues in the parameter space $\mu_1 - \mu_3$



$\mu_5 = 0.0008$



$\mu_5 = 0.00111$

Figure4: multi stability of PMSM in phase plan

Figure3 explain the evolution of the surface of the real and imaginary eigenvalues in the parameter plan. Then all point in the surface may verify the two conditions $F(x^e) = 0$, thus the real eigenvalues cross a zero for $\mu_1 = \mu_3 = 0$. Now, if a small perturbation of the parameter μ_5, the dynamic of the PMSM drive will be changed its structure, see figure4. And then, the multi stability machine will be defined by to equilibrium point EP_0 and EP_1 for $\mu_5 = 0.00111$.

IV-Conclusion

The paper presents a through bifurcation analysis of detailed permanent magnet machine with feedback drive, showing the effect of different control parameters and on the bifurcation and associated stability of the system. This paper concentrates on showing the practical applications of the bifurcation theory in order to investigate the real stability of machine

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